Simple Waves in Saturated Porous Media*
(II. The Non Isothermal Case)

S. SOREK**, A. KRYLOV***, A. LEVY****
and G. BEN-DOR****

Wave equations may be invoked when an abrupt change in pressure applied to a compressible fluid in saturated porous media[1] (Bear et al. Fluid Dynamics Research, Vol. 9, pp. 155-164). This paper presents a method leading to generalized fluid density, pressure and temperature. Using these generalized characteristics, we write Euler's equation as the one-dimensional expression for the analytical solution of equation of motion of the fluid. To obtain this, equation the porosity and the matrix strain and temperature of the solid are developed as explicit functions of pressure.

Key Words: Ideal Compressible Fluid, Saturated Porous Media, Homogeneous, Isotropic, Thermoelastic Solid Matrix, Macroscopic Balance Equations.

1. Introduction

The aim of the present study was to develop a simple solution to the pressure wave problem as experienced by a compressible fluid in saturated porous media, following an abrupt pressure change. This motion equation was originally described by Bear and Sorek[1] for the case of a deformable isothermal solid matrix. Further developments for the case of deformable non isothermal conditions were introduced by Bear et al.[2] and Sorek et al.[3] Levy et al.[4] considered the case of abrupt simultaneous changes of both the pressure and the fluid temperature. A macroscopic one-dimensional analytical solution was developed and described in part I of this paper for the case concerning isothermal conditions [Krylov et al.[5]]. That paper also lists some of the recent publications in the field of linear waves in porous media. A rather extensive literature survey was also presented by Corapcioglu[6].

In the present study, a macroscopic one-dimensional analytical model concerning non isothermal conditions is developed.

2. The Conceptual Model

The conceptual model is based on the following set of simplifying assumptions:

[A. 1] The fluid is ideal and compressible; it is a perfect gas.

[A. 2] The solid phase preserves its volume.

[A. 3] The solid matrix is thermoelastic, and is assumed to undergo only small deformations.

[A. 4] The dispersive flux of momentum is much smaller than its advective one and may, therefore, be neglected.

[A. 5] The dispersive and diffusive mass fluxes of

* Received 11th January, 1995.

** Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva, Israel also affiliated with the Water Resource Research Center, J. Blaustein Desert Research Institute, Sede Boker Campus 84990, Israel

*** Visiting Professor from the Institute for Earth Physics of the Russian Academy of Science, Moscow, Russia

**** Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva, Israel

the fluid, and the dispersive flux of the solid, are much smaller than the corresponding advective ones and may, therefore, be neglected.

[A. 6] The microscopic solid-fluid interfaces are material surfaces (i.e., the fluid cannot penetrate the solid).

[A. 7] The stress-strain relationships for the solid, at the microscopic level, and for the solid matrix, at the macroscopic level, have the same form.

[A. 8] Energy processes for the fluid and for the solid are assumed to be reversible, and the specfic heats at constant volume, c_{fv}^*, and at constant pressure, c_{fp}^*, for the fluid and that at constant strain, c_s^*, for the solid, are assumed to be constant.

[A. 9] There are no external energy sources.

[A. 10] The energy associated with viscous dissipation is assumed to be negligible.

[A. 11] The conductive and dispersive heat fluxes of the phases are assumed to be negligible when compared to their advective heat flux.

[A. 12] The rate of heat transferred between the fluid and solid phases is assumed to be negligible.

3. Macroscopic Balance Equations

Based on Bear et al.[2] and Sorek et al.[3] together with assumptions [A. 1], [A. 4], [A. 5] and [A. 6], it can be shown that at a certain time interval, following an abrupt pressure change, the equation of motion of the fluid conforms to that of a wave equation. The fluid mass balance equation.

$$\frac{\partial}{\partial t}(\phi\rho_f)+\frac{\partial}{\partial x}(\phi\rho_f u_f)=0 \qquad (1)$$

in which ϕ is the porosity, u_f is the velocity of the fluid and its density, ρ_f, is given by the equation of state of the fluid.

$$\rho_f=\rho_{f(T_f,\,p)} \qquad (2)$$

Here p is the pressure of the fluid and T_f is its temperature. The fluid momentum balance equation is given by.

$$\frac{\partial u_f}{\partial t}+u_f\frac{\partial u_f}{\partial x}+\frac{1}{\rho_f}\frac{\partial p}{\partial x}T^*=0 \qquad (3)$$

in which T^* is the tortuosity coefficient.

Following assumptions [A. 8] to [A. 12] the fluid energy balance equation is

$$\frac{\partial}{\partial t}\left(\phi\rho_f c_{fv}^* T_f\right)+\frac{\partial}{\partial x}(\phi\rho_f c_{fv}^* T_f u_f)+T_f\frac{\beta_T}{\beta_p}\Big|_{\rho_f}\frac{\partial}{\partial x}u_f=0 \qquad (4)$$

where c_{fv}^* and T_f are the specific heat at constant volume and the temperature of the fluid phase, respectively. Recall that according to assumption [A. 8] c_{fv}^* is constant.

The coefficient of fluid volumetric thermal expansion, β_T, is defined by

$$\beta_T\equiv\frac{1}{\rho_f}\frac{\partial\rho_f}{\partial T_f}\Big|_p \qquad (5)$$

and β_p, the compressibility of the fluid is defined by

$$\beta_p\equiv\frac{1}{\rho_f}\frac{\partial\rho_f}{\partial p}\Big|_{T_f}. \qquad (6)$$

Note that with assumption [A. 12] we regard the motion as being adiabatic throughout every point in the fluid. In such a case the specific entropy, s_f, of the fluid remains constant. We will thus exchange Eq. (4) with an equivalent form for the fluid energy balance equation (see Appendix I).

$$\frac{\partial}{\partial t}(\phi\rho_f s_f)+\frac{\partial}{\partial x}(\phi\rho_f s_f u_f)=0 \qquad (7.1)$$

Assumptions [A. 10] and [A. 11] refer specifically to fluid in which thermal conductivity and viscosity are unimportant, in such cases the processes of energy dissipation are negligible. These are the *ideal* fluids, the motion of which must necessarily be adiabatic. Hence, Eq. (7. 1) is the general macroscopic equation describing adiabatic motion of an ideal fluid.

We have 6 variables so far ϕ, ρ_f, p, T_f, u_f and s_f. To solve for these variables, we have 3 balance equations (1), (3), (7. 1) and a constitutive relation (2). To address this difference, we add an additional constitutive relation. We assume that the macroscopic relationships of the fluid constitutive laws are similar in their form to their microscopic counterparts. Thus, for an ideal fluid we add the relation

$$\left(\frac{p}{p_0}\right)\left(\frac{\rho_{f0}}{\rho_f}\right)^{c_{fv}^*/c_{fp}^*}=\exp\left[\frac{s_f-s_{f0}}{c_{fv}^*}\right] \qquad (7.2)$$

where at a given reference state $p=p_0$, $\rho_f=\rho_{f0}$ and $s_f=s_{f0}$.

We also express Eq. (2) in its explicit form assuming a perfect gas behavior, so that

$$\rho_f=\frac{p}{RT_f} \qquad (7.3)$$

in which R is the specific gas constant.

We now 5 variables, ϕ, ρ_f, p, u_f and s_f, and only 4 expressions, (1), (3), (7.1) and (7.2).

To find a relation for the porosity, ϕ, we will later address approximate forms of the solid balance equations.

Following Bear et al.[2], Sorek et al.[3], Levy et al.[4] and Krylov et al.[5] and in accordance with assumptions [A. 2] to [A. 9], [A. 11] and [A. 12] the solid mass balance equation is

$$\frac{\partial\phi}{\partial t}=(1-\phi)\frac{\partial u_s}{\partial x} \qquad (8)$$

in which u_s is the velocity of the solid. The momentum balance equation of the solid is given by

$$\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}-\frac{\lambda_s}{\rho_s(1-\phi)}\frac{\partial^2 w}{\partial x^2}+\frac{\eta}{\rho_s(1-\phi)}\frac{\partial T_s}{\partial x}+\frac{T^*}{\rho_s}\frac{\partial p}{\partial x}=0 \qquad (9)$$

where λ_s is the Lamé constant of the elastic solid, ρ_s

its constant density, w the solid skeleton displacement and η the constant coefficient of thermal stress.

The solid energy balance equation is given by

$$c_s^*\rho_s(1-\phi)\left[\frac{\partial T_s}{\partial t}+u_s\frac{\partial T_s}{\partial x}\right]+\eta T_s\frac{\partial u_s}{\partial x}=0 \quad (10)$$

where c_s^* and T_s are the specific heat and temperature of the solid phase, respectively. Recall that according to assumption [A. 8], c_s^* is constant.

In view of assumptions [A. 11] and [A. 12] the solid-phase motion is adiabatic.

4. Approximate Forms of the Balance Equations of the Solid

Let us now consider the Struhal number, St^e, associated with an intensive quantity e. Following the discussion in part I of this paper [Krylov et al.[5]] which led to Eq. (8), we assume (in the case $e\equiv T_s$)

$$St^{T_s}\equiv\frac{L_c^{T_s}}{t_c^{T_s}u_c^s}\gg 1 \quad (11)$$

in which L_c^e and t_c^e are the characteristic spatial increment and time step associated with e respectively, and u_c^s is the characteristic velocity of the solid phase.

It was pointed out in part I of this paper [Krylov et al.[5]], that the mass balance of the solid can be written in the form

$$\frac{1}{1-\phi}\frac{\partial\phi}{\partial t}=\frac{\partial u_s}{\partial x}=\frac{\partial\varepsilon}{\partial t} \quad (12)$$

where we define u_s by

$$u_s\cong\frac{\partial w}{\partial t} \quad (13)$$

and use the compatibility equation to write the skeleton strain, ε, of the solid in the form

$$\varepsilon=\frac{\partial w}{\partial x}. \quad (14)$$

This strain was shown to have an order of magnitude of

$$\varepsilon\ll 1 \quad (15)$$

which allowed, us, by integration of Eq. (12), to write

$$\frac{1-\phi}{1-\phi_0}=\exp(-\varepsilon)\cong 1-\varepsilon \quad (16)$$

where we had assumed that $\varepsilon(x,0)=0$ and $\phi(x,0)=\phi_0$.

Hence, in view of Eqs. (11), (14) and (16), we rewrite Eq. (10) in the form

$$c_s^*\rho_s(1-\phi_0)\frac{\partial T_s/\partial t}{T_s}+\eta\frac{\partial\varepsilon/\partial t}{1-\varepsilon}=0. \quad (17)$$

The solution of Eq. (17) is thus

$$T_s=T_{s_0}(1-\varepsilon)^{\eta/c_s^*\rho_s(1-\phi_0)} \quad (18)$$

in which T_{s_0} is the solid temperature at $\varepsilon=0$. In view of Eq. (15), Eq. (18) can be simplified to read

$$T_s=T_{s_0}\left[1-\frac{\eta\varepsilon}{c_s^*\rho_s(1-\phi_0)}\right]. \quad (18.a)$$

We now consider the assumption

$$St^w\equiv\frac{L_c^w}{t_c^w u_c^s}\gg 1 \quad (19)$$

which is similar to that used in deriving Eq. (11). Hence by virtue of Eq. (13) and (19) we rewrite Eq. (9), the solid momentum balance equation, in the form

$$\frac{\partial^2 w}{\partial t^2}-c_s^2\frac{\partial^2 w}{\partial x^2}+\frac{\eta}{\rho_s(1-\phi)}\frac{\partial T_s}{\partial x}+\frac{T^*}{\rho_s}\frac{\partial p}{\partial x}=0 \quad (20)$$

where c_s the propagation speed of the displacement wave in the solid phase is given by

$$c_s^2\equiv\frac{\lambda_s}{\rho_s(1-\phi)}. \quad (21)$$

Let us now assume that

$$St^w M_s\ll 1 \quad (22)$$

where $M_s(\equiv u_s/c_s)$ is the solid Mach number, In view of Eq. (19) and (22) we note that $M_s\ll 1$.

Following Eq. (14) and (22) we can approximate the solid momentum balance equation appearing in Eq. (20) as

$$-\frac{\lambda_s}{(1-\phi)}\frac{\partial\varepsilon}{\partial x}+\frac{\eta}{1-\phi}\frac{\partial T_s}{\partial x}+T^*\frac{\partial p}{\partial x}=0. \quad (24)$$

Substituting Eqs. (16) and (18) into Eq. (24) yields

$$\frac{\lambda_{eff}}{(1-\phi_0)(1-\varepsilon)}\frac{\partial(1-\varepsilon)}{\partial x}=T^*\frac{\partial p}{\partial X} \quad (25)$$

in which

$$\lambda_{eff}\equiv\frac{T_{s_0}\eta^2}{c_s^*\rho_s(1-\phi_0)}+\lambda_s. \quad (26)$$

Solving Eq. (25), while assuming that $p=p_0$ at $\varepsilon=0$, yields

$$(1-\varepsilon)=\exp\left[\frac{-T^*(1-\phi_0)}{\lambda_{eff}}(p-p_0)\right]. \quad (27)$$

However, using Eq. (15) we may expand Eq. (27) to read

$$\varepsilon\cong\frac{T^*(1-\phi_0)}{\lambda_{eff}}(p-p_0). \quad (28)$$

Substituting Eq. (28) into Eq. (16) yields

$$\phi=\phi_0+\frac{(1-\phi_0)^2T^*}{\lambda_{eff}}(p-p_0). \quad (29)$$

Accordingly, using Eq. (29), we may rewrite Eq. (18) in the form

$$T_s=T_{s_0}\left[1-\frac{\eta T^*}{c_s^*\rho_s\lambda_{eff}}(p-p_0)\right]. \quad (30)$$

Using the compatibility equation expressed in Eq. (14) and in light of Eq. (28), we write

$$w=\frac{1-\phi_0}{\lambda_{eff}}T^*\int_{-\infty}^{x}(p-p_0)dx \quad (31)$$

where we have assumed that $w_{(-\infty,t)}=0$.

By combining Eq. (13) with Eq. (31) we obtain

$$u_s=\frac{1-\phi_0}{\lambda_{eff}}T^*\int_{-\infty}^{x}\frac{\partial}{\partial t}(p-p_0)dx. \quad (32)$$

Equation (29) describes the porosity as a function of the initial solid temperature and pressure; we now aim at writing a simple form for the solution of the fluid energy and motion balance equations.

5. Approximate Forms of the Fluid Balance Equations

Equations (1), (3), (7.1), (7.2) and (29) allow

us to solve for the five variables ϕ, ρ_f, u_f, p and s_f. To rewrite Eq. (3) to an equivalent Euler equation, we introduce new variables, ρ_n and p_n, as density and pressure, respectively, of a generalized fluid. These are defined by

$$\rho_n=\phi\rho_f \tag{33}$$

and

$$dp_n=T^*\phi dp. \tag{34}$$

Substituting Eq.(29) into Eq. (34) and integrating, yields

$$\left(p-p_0+\frac{A+\sqrt{2Bp_n+A^2}}{B}\right)\left(p-p_0+\frac{A-\sqrt{2Bp_n+A^2}}{B}\right)=0 \tag{35.1}$$

where

$$A\equiv\phi_0 T^* \tag{35.2}$$

and

$$B\equiv\frac{[(1-\phi_0)T^*]^2}{\lambda_{eff}}. \tag{35.3}$$

In view of Eq.(35) we note that Eq.(35.1) has to be in the form

$$p_n=\left[A+\frac{B}{2}(p-p_0)\right](p-p_0). \tag{36}$$

Hence, in view of Eqs.(29), (35) and (36), we write ϕ in terms of p_n, namely

$$\phi=\frac{\sqrt{A^2+2Bp_n}}{T^*}. \tag{37}$$

Thus, by virtue of Eqs.(33), (36) and (37) we rewrite Eq.(7.2) in terms of ρ_n and p_n and obtain the following:

$$\left(\frac{\sqrt{A^2+2Bp_n}-A}{Bp_0}\right)\left(\frac{\sqrt{A^2+2Bp_n}}{T^*}\frac{\rho_{f_0}}{\rho_n}\right)^{c^*_{fp}/c^*_{fv}}=\exp\left(\frac{s_f-s_{f0}}{c^*_{fv}}\right). \tag{38}$$

Multiplying Eq.(1) by s_f and subtracting it from Eq. (7.1), yields

$$\frac{\partial s_f}{\partial t}+u_f\frac{\partial s_f}{\partial x}=0. \tag{39}$$

Let us define the hydrodynamic time derivative by

$$\frac{D_f}{Dt}\equiv\frac{\partial}{\partial t}+u_f\frac{\partial}{\partial x}. \tag{40}$$

In view of Eq.(40), we may rewrite Eqs.(37) to (39), the generalized fluid mass, momentum and energy balance equations, in the form

$$\frac{D_f\rho_n}{Dt}+\rho_n\frac{\partial u_f}{\partial x}=0 \tag{41}$$

$$\frac{D_f u_f}{Dt}+\frac{1}{\rho_n}\frac{\partial p_n}{\partial x}=0 \tag{42}$$

$$\frac{D_f s_f}{Dt}=0. \tag{43}$$

We now have four variables: ρ_n, u_f, p_n and s_f. To solve for these variables, we have three balance equations (41) to (43) and a constitutive relation (38).

In the following an expression for the temperature, T_n, of the generalized fluid is developed. This will enable us to write the generalized state function commencing from (7.3).

We start by introducing the relation for the generalized enthalpy, $h_n(p_n, s_f)$. Using the definition for the enthalpy,

$$dh_{(p,\,s_f)}=T_f ds_f+\frac{dp}{\rho}, \tag{44}$$

we postulate that

$$dh_{n(p_n,\,s_f)}=T_n ds_f+\frac{dp_n}{\rho_n}. \tag{45}$$

From Eq. (48) we have

$$\left(\frac{1}{\rho_n}\right)=\frac{\partial h_n}{\partial p_n} \tag{46}$$

and

$$T_n=\frac{\partial h_n}{\partial s_f}. \tag{47}$$

By virtue of Eqs.(33), (34) and (44) we write

$$\left(\frac{1}{\rho_n}\right)=\left(\frac{1/\rho_f}{\phi}\right)=\frac{1}{\phi}\frac{\partial h}{\partial p}=T^*\frac{\partial h}{\partial p_n}. \tag{48}$$

By comparing Eq.(49) to Eq. (51) we find

$$h_{n(p_n,\,s_f)}=T^* h_{(p,\,s_f)}. \tag{49}$$

In view of Eqs.(44), (47) and (49) we obtain

$$T_n=T^* T_f. \tag{50}$$

The equation of state of the generalized fluid may now be obtained from Eqs.(33), (36), (37) and (50) when substituted into Eq.(7.3)

$$\rho_n=\frac{\sqrt{A^2+2Bp_n}(Bp_0-A+\sqrt{A^2+2Bp_n})}{BRT_n}. \tag{51}$$

Following assumption [A. 3] for thermoelastic rigid materials (e.g., SiC or Al_2O_3) we find that $\lambda_s=$ 400 GPa. The thermal stress coefficient, η, may be related to this λ_s by

$$\eta=\alpha\lambda_s \tag{52}$$

in which α is the thermal expansion coefficient. In view of Eq.(52) we rewrite Eq.(26) to read

$$\lambda_{eff}=\left[1+\frac{T_{s_0}\alpha^2\lambda_s}{c^*_s\rho_s(1-\phi_0)}\right]\lambda_s. \tag{53}$$

Considering orders of magnitudes, for example, $T_{s_0}=300$ K, $\alpha=5\cdot10^{-6}\,K^{-1}$, $\lambda_s=400$ GPa, $c^*_s=1\,000$ J/kg/K, $\rho_s=2\,000$ kg/m^3, and $0<\phi_0<1$, we can rewrite Eq.(53) by its approximate form

$$\lambda_{eff}\cong\lambda_s. \tag{54}$$

Accordingly, in view of Eq.(54) for $\lambda_s=O(400$ GPa) we find B of Eq.(35.3) to be

$$B\ll1. \tag{55}$$

By virtue of Eqs.(35.2) and (55), we rewrite Eq. (36) to read

$$p_n\cong\phi_0 T^*(p-p_0) \tag{56}$$

and Eq.(40), to read

$$\phi\cong\phi_0. \tag{57}$$

Note that Eqs.(56) and (57) are in agreement with the isothermal case [Krylov et al.[5]]. Furthermore, introducing Eq.(55) into Eq.(51) yields an approximate relation of the form

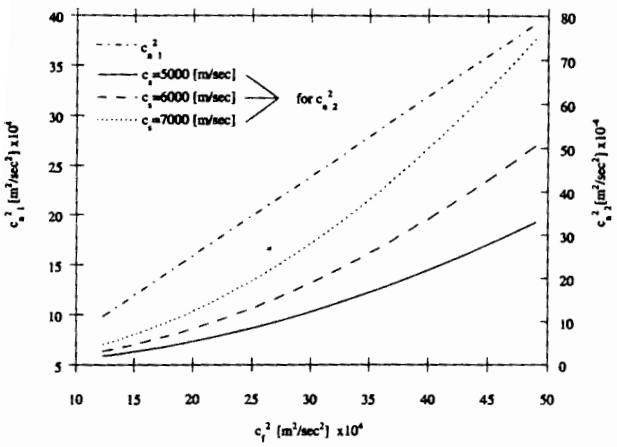

Fig. 1 Sound speed of the 'new' fluid, $c_n(c_n^2=c_{n1}^2+c_{n2}^2\cong c_{n1}^2)$, and of the original one c_f. ($T^*=0.8$, $\lambda_s=400$[GPa], $\rho_s=2\,000$[kg/m³])

$$\rho_n\cong\frac{\phi_0 p_0 T^*}{RT_n}. \tag{58}$$

Hence we note that ρ_n (for the generalized fluid) becomes dependent only on the temperature. Combining Eqs.(30), (52) and (54) results in

$$\left(\frac{T_s}{T_{s_0}}-1\right)=-\frac{\alpha T^*}{c_s^*\rho_s}(p-p_0). \tag{59}$$

This implies a linear relation with pressure drop. However, when this relation is plotted (see Fig. 1) for characteristic values of α, c_s^* and three different values of ρ_s it is evident that

$$T_s\cong T_{s_0}. \tag{60}$$

6. Conclusion

Abrupt simultaneous changes in the pressure and the temperature in saturated deformable porous media result in nonlinear wave equations of the momentum balance equations of the phases [Levy et al.[4]]. An analytical one-dimensional solution for the resulting wave equations under non-isothermal conditions has been developed. It was found that the solution conforms to the case of simple (Riemann) waves while the energy balance equation conforms to an entropy flux equation. The balance equations apply to a generalized fluid. For a rigid thermoelastic solid matrix, it was proven that the approximate generalized pressure is linearly dependent on the original pressure drop and the porosity can be regarded as constant. These findings are in agreement with the isothermal case. The temperature of the generalized fluid is linearly dependent on that of the original fluid. The approximate generalized equation of state disregards pressure and relates between the generalized fluid density and its temperature.

Appendix I :

It is our aim to show that when concerning an ideal fluid we can replace between its energy balance equation, Eq.(4), by its entropy flux equation, Eq. (7.1).

Let us multiply Eq.(1), the mass balance equation of the fluid, by $c_{fv}^* T_f$ and subtract it from Eq. (4). This gives

$$c_{fv}^*\frac{D_f T_f}{Dt}+\frac{T_f}{\rho_f}\frac{\beta_T}{\beta_p}\Big|_{\rho_f}\frac{\partial u_f}{\partial x}=0. \tag{I.1}$$

For the case of an ideal fluid, under assumption [A. 1], and in view of Eq. (7. 3), we rewrite its equation of state in the form

$$\frac{1}{\rho_f}\frac{\beta_T}{\beta_p}\Big|_{\rho_f}=R. \tag{I.2}$$

Hence, by virtue of Eq.(I.2) we rewrite Eq.(I.1) in the form

$$c_{fv}^*\frac{D_f T_f}{Dt}+RT_f\frac{\partial u_f}{\partial x}=0. \tag{I.3}$$

The entropy flux equation may also be rewritten. If we subtract Eq.(1) from Eq.(7.1), we obtain

$$\frac{D_f s_f}{Dt}=0. \tag{I.4}$$

Hence, we are left to show that Eqs.(I.3) and (I.4) are interchangeable.

For a perfect gas, we have

$$de_f=c_{fv}^* dT_f \tag{I.5}$$

in which, e_f is the internal specific energy of the gas.

Accounting for assumptions [A. 9] and [A. 12], the characteristic feature of the adiabatic process is that heat is not being absorbed. Hence, we may write

$$0=de_f+pd(1/\rho_f). \tag{I.6}$$

In the case of an ideal fluid, in view of Eqs.(7.3), (I. 5) and (I.6), we write the Clasius specific entropy function, in the form

$$s_f=c_{fv}^* ln\, T_f+(Rln(1/\rho_f). \tag{I.7}$$

Thus, by virtue of Eqs.(7.3), (I.6) and (I.7), we write

$$\frac{D_f s_f}{Dt}=\frac{c_{fv}^*}{T_f}\frac{D_f T_f}{Dt}+\frac{\partial p}{\partial T_f}\frac{D_f(1/\rho_f)}{Dt}=0. \tag{I.8}$$

Expanding the second R. H. S. term of Eq.(I.8) and making use of Eqs.(1) and (7.3), we obtain

$$\frac{\partial p}{\partial T_f}\frac{D_f(1/\rho_f)}{Dt}=R\rho_f\left(\frac{-1}{\rho_f^2}\right)\frac{D_f(\rho_f)}{Dt}=R\frac{\partial u_f}{\partial x}. \tag{I.9}$$

Hence, substituting Eq.(I.9) into Eq.(I.8) finally yields

$$\frac{D_f s_f}{Dt}=c_{fv}^*\frac{D_f T_f}{Dt}+RT_f\frac{\partial u_f}{\partial x}=0. \tag{I.10}$$

References

(1) Bear, J. and Sorek, S. Evolution of Governing Mass and Momentum Balances Following an Abrupt Pressure Impact in Porous Medium, Transport in Porous Media, Vol. 5 (1990), p. 169.

(2) Bear, J., Sorek, S., Ben-Dor, G. and Mazor, G., Displacement Waves in Saturated Thermoelastic Porous Media. I. Basic Equations, Fluid Dynamic Research, Vol. 9 (1992), p. 155.
(3) Sorek, S., Bear, J., Ben-Dor, G. and Mazor, G., Shock Waves in Saturated Thermoelastic Porous Media, Transport in Porous Media, Vol. 9 (1992), p. 3.
(4) Levy, A., Sorek, S., Ben-Dor, G. and Bear, J., Evolution of the Balance Equations in Saturated Thermoelastic Porous Media Following Abrupt Simultaneous Changes in Pressure and Temperature, Transport in Porous Media, Vol. 21 (1996), p. 241.
(5) Krylov, A., Sorek, S., Levy, A. and Ben-Dor, G., Simple Waves in Saturated Porous Media. I. The Isothermal Case, JSME Int. J., Ser. B, Vol. 39, No. 2 (1996), p. 294.
(6) Corapcioglu, M. Y., Wave Propagation in Porous Media - a Review, in J. Bear and M. Y. Corapcioglu (eds.) Transport Processes in Porous Media, (1991), p. 373, Kluwer Acad. Pubs. Dordrect.